

## Solutions

Name: \_\_\_\_\_

This assignment consists of seven questions, each worth five points for a total of 35 points. To receive full credit you must **show all necessary work**. You should write your answers in the spaces provided, but if you require more space please *staple any extra sheets* you use to this assignment. If you are having trouble with any of the problems, look at the lecture notes and exercises in the lecture notes for help. Remember to start this assignment early, your next quiz is based on this assignment.

1. A function  $f(x)$  crosses the points  $(1, 300)$  and  $(3, 243)$ . Find a formula for  $f(x)$  if

(a)  $f(x)$  is a linear function.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{243 - 300}{3 - 1} = -\frac{57}{2}$$

$$y - y_1 = m(x - x_1)$$

$$f(x) - f(3) = -\frac{57}{2}(x - 3)$$

$$\text{Answer: } f(x) = -\frac{57}{2}(x - 3) + 243$$

(b)  $f(x)$  is an exponential function.

$$\frac{f(3)}{f(1)} = \frac{P_0 a^3}{P_0 a^1} = a^2 = \frac{243}{300} = 0.81$$

$$\Rightarrow a = 0.9$$

$$f(1) = P_0 (0.9)^1 = 300$$

$$\Rightarrow P_0 = \frac{300}{0.9} = 333.33$$

$$\text{Answer: } f(x) = 333.33(0.9)^x$$

2. A certain company manufactures chairs. The cost to produce 265 chairs is \$11,627 and the cost to produce 560 chairs is \$21,592.

(a) Find a cost function  $c(q)$  representing the cost of producing  $q$  chairs.

$$m = \frac{21592 - 11627}{560 - 265} = \frac{9965}{295}$$

$$= \frac{1993}{59}$$

$$(265, 11627) \quad (560, 21592)$$

$$y - y_1 = m(x - x_1)$$

$$C(q) - C(265) = \frac{1993}{59}(q - 265)$$

$$\text{Answer: } C(q) = \frac{1993}{59}q + \frac{157848}{59}$$

(b) Find the profit function  $\pi(q)$  representing the profit made from selling  $q$  chairs, if each chair sells for \$79.

$$\text{Answer: } \pi(q) = \frac{2668}{59}q - \frac{157848}{59}$$

(c) Find the break-even point for this company.

$$\frac{2668}{59q} = \frac{39462}{667} \Rightarrow q = 59.16$$

Answer: 60

3. Consider the function  $P(t) = 58(1.76)^t$ .

(a) Use words or an expression to give the meaning of  $\ln(1.76)$ .

$$e^{\ln(1.76)} = 1.76 \quad \text{or} \quad \text{The power to which } e \text{ should be raised to equal } 1.76$$

(b) Write  $P(t)$  in the form  $P(t) = P_0 e^{kt}$ .

$$P(t) = 58 e^{\ln(1.76)t}$$

Answer: \_\_\_\_\_

(c) What is the continuous growth factor of  $P(t)$ ?

$$\ln(1.76)$$

Answer: \_\_\_\_\_

4. There is 500mg of a certain drug in a person's bloodstream at time  $t = 0$ . This amount is decreasing at a rate of 34% per hour.

(a) How much of the drug is left after 1 hour?

$$330 \text{ mg}$$

Answer: \_\_\_\_\_

(b) How much of the drug is left after 2 hours?

$$217.8$$

Answer: \_\_\_\_\_

(c) Find a formula for the amount of this drug left in the body at time  $t$  hours if the drug is leaving the body;

i. at an hourly rate.

$$A(t) = 500(0.66)^t$$

Answer: \_\_\_\_\_

ii. continuously. ~~(i) at an hourly rate.~~

$$A(t) = 500 e^{-0.34t}$$

Answer: \_\_\_\_\_

Cont.

5. Harry and Ron invest in different companies at the same time,  $t = 0$ . Harry invested \$12,763 and Ron invested \$8,941. After time  $t = 7$  the value of Harry's investment had risen to \$14,129 and Ron's had risen to \$9,857.

(a) Find the average rate of change of Harry's investment.

$$\frac{14129 - 12763}{7 - 0} = \frac{1366}{7} \approx 195.14$$

Answer: \$195.14

(b) Find the average rate of change of Ron's investment.

$$\frac{9857 - 8941}{7 - 0} = \frac{916}{7} \approx 130.86$$

Answer: \$130.86

(c) Whose increase in the value of their investment was more significant?

$$\text{Harry } \frac{\left(\frac{1366}{7}\right)}{12763} = 0.01529\% \quad \text{Ron } \frac{\left(\frac{916}{7}\right)}{8941} = 0.01464\%$$

Answer: Harry

6. A charity is trying to raise money by selling cupcakes. When each cupcake costs \$1, they manage to sell 175 cupcakes. When the price goes up to \$1.25 per cupcake, they only sell 154 cupcakes.

(a) Find the relative change in the price of a cupcake.

$$\frac{1.25 - 1}{1} = 0.25$$

Answer: 25%

(b) Find the relative change in the quantity of cupcakes sold.

$$\frac{154 - 175}{175} = -0.12$$

Answer: -12%

(c) Find the relative change in revenue.

$$\frac{1.25(154) - 175}{175} = 0.1$$

Answer: 10%

Cont.

7. Read the section on Distance, Velocity and Speed on page 22 of the lecture notes.

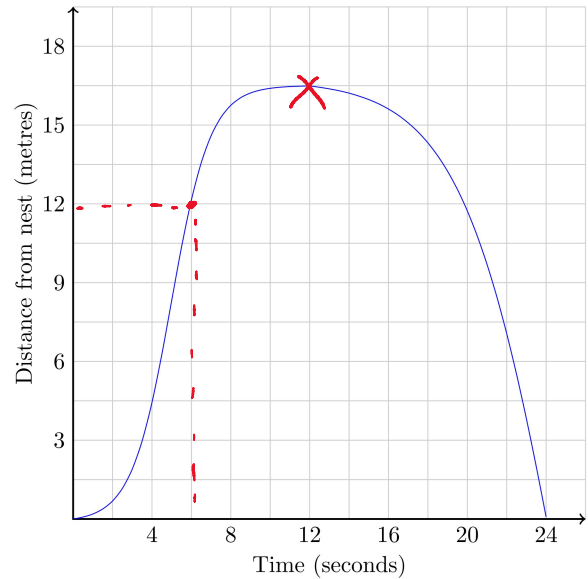
A bird takes off from its nest, heading north in a straight line. The distance, in metres, from its nest is given by the function graphed below.

- (a) At what time does the bird change direction and head south, back towards its nest?

Answer: 12 secs.

- (b) How far away was the bird from its nest by the time it turned around?

Answer: 16.5m



- (c) What was the birds average velocity from the time it left its nest to the time it turned around?

$$\frac{16.5}{12} = \frac{11}{8} = 1.375$$

1.375 m/s

Answer: \_\_\_\_\_

- (d) What was the the birds average velocity from  $t = 6$  to the time the bird flew back to its nest?

$$\frac{0-12}{24-6} = \frac{-12}{18} = -\frac{2}{3}$$

-0.667 m/s

Answer: \_\_\_\_\_

- (e) What was the the birds average *speed* over this interval?

0.667 m/s

Answer: \_\_\_\_\_

The End.